[Answer Key]

ACT 2020, FINAL EXAMINATION
ECONOMIC AND FINANCIAL APPLICATIONS
APRIL 16, 2007
9:00AM - 11:00AM
University Centre RM 210 - 224 (Seats 285 - 329)

Instructor: Hal W. Pedersen

You have 120 minutes to complete this examination. When the invigilator instructs you to stop writing you must do so immediately. If you do not abide by this instruction you will be penalised.

Each question is worth 10 points. If the question has multiple parts, the parts are equally weighted unless indicated to the contrary. Provide sufficient reasoning to back up your answer but do not write more than necessary.

This examination consists of 12 questions. Answer each question on a separate page of the exam book. Write your name and student number on each exam book that you use to answer the questions. Good luck!

Q1: [Text Q1.7 pg. 18]

Suppose you desire to short-sell 400 shares of JKI stock, which has a bid price of \$25.12 and an ask price of \$25.31. You cover the short position 180 days later when the bid price is \$22.87 and the ask price is \$23.06.

[3pts] **a.** Taking into account only the bid and ask prices (ignoring commissions and interest), what profit did you earn?

[4 pts] **b.** Suppose that there is a 0.3% commission to engage in the short-sale (this is the commission to sell the stock) and a 0.3% commission to close the short-sale (this is the commission to buy the stock back). How do these commissions change the profit in the previous answer?

[3 pts] **c.** Suppose the 6-month interest rate is 3% and that you are paid nothing on the short-sale proceeds. How much interest do you lose during the 6 months in which you have the short position?

Q2: [Text Q 1.3 pg. 17]

ABC stock has a bid price of \$40.95 and an ask price of \$41.05. Assume there is a \$20 brokerage commission.

[3 pts] **a.** What amount will you pay to buy 100 shares?

[3 pts] **b.** What amount will you receive for selling 100 shares?

[4 pts] **c.** Suppose you buy 100 shares, then immediately sell 100 shares with the bid and ask prices being the same in both cases. What is your round-trip transaction cost?

Q3: [Q 2.9 pg. 55 text.]

An *off-market* forward contract is a forward where either you have to pay a premium or you receive a premium for entering into the contract. (With a standard forward contract, the premium is zero.) Suppose the effective annual interest rate is 10% and the S&R index is 1000. Consider 1-year forward contracts.

(1) [5 pts] Suppose you are offered a long forward contract at a forward price of \$1200. How much would you need to be paid to enter into this contract?

(2) [5 pts] Suppose you are offered a long forward contract at \$1000. What would you be willing to pay to enter into this forward contract?

[Assume "arbitrage-free" market.]

Q4: [Practice Problems Ch. 2 #4.]

Suppose MNO stock pays no dividends and has a current price of \$150. The forward price for delivery in one year is \$157.50. Suppose the one-year effective annual interest rate is 5%.

[3 pts] (a) Graph the payoff and profit diagrams for a short forward contract on MNO stock with a forward price of \$157.50.

[3 pts] (b) Is there any advantage to short selling the stock or selling the forward contract?

[4 pts] (c) Suppose MNO paid a dividend of \$3 per year and everything else stayed the same. Is there any advantage to selling the forward contract?

[Provide justification for your answers.]

For Q5 + Q6, assume the effective 6-month interest rate is 2%, the S&R 6-month forward price is $1020, and use these premiums for S&R options with 6 months to expiration:

Strike	Call	Put
$950	$120.405	$51.777
1000	93.809	74.201
1020	84.470	84.470
1050	71.802	101.214
1107	51.873	137.167

Q5: [Text 3.8]

Suppose the premium on a 6-month S&R call is $109.20 and the premium on a put with the same strike price is $60.18. What is the strike price?

Q6: [Text 3.12]

Suppose you invest in the S&R index for $1000, buy a 950-strike put, and sell a 1107-strike call. Draw a profit diagram for this position.

[Clearly label your chart to show where your profit curve cuts the y-axis.]

Q7: [Text 4.16]

Suppose that firms face a 40% income tax rate on all profits. In particular, losses receive full credit. Firm A has a 50% probability of a $1000 profit and a 50% probability of a $600 loss each year. Firm B has a 50% probability of a $300 profit and a 50% probability of a $100 profit each year.

a. What is the expected pre-tax profit next year for firms A and B?

b. What is the expected after-tax profit next year for firms A and B?

Q8:

KidCo Cereal Company sells "Sugar Corns" for $2.50 per box. The company will need to buy 20,000 bushels of corn in 6 months to produce 40,000 boxes of cereal. Non-corn costs total $60,000. What is the company's profit if they purchase call options at $0.12 per bushel with a strike price of $1.60? Assume the 6-month interest rate is 4.0% and the spot price in 6 months is $1.65 per bushel.

Q9: [Practice Problem 5.5.]

The MNO stock trades at $66, the risk-free rate is 5%, and the MNO stock pays constant quarterly dividends of $0.45. Assume that the first dividend is coming three months from today, and the last one coming immediately before the expiration of the forward contract. You can trade MNO single stock futures with an expiration of nine months.

Suppose you observe a 9-month forward price of $67.50. What arbitrage would you undertake?

[You must explicitly describe what you will trade and what your cash flows will be at all relevant points in time.]

Q10: [Text 5.15]

Suppose the S&R index is 800, and that the dividend yield is 0. You are an arbitrageur with a continuously compounded borrowing rate of 5.5% and a continuously compounded lending rate of 5%.

(1) Supposing that there are no transaction fees a cash-and-carry arbitrage is not profitable if the forward price is less than A.

A reverse cash-and-carry arbitrage is not profitable if the forward price is greater than B.

Compute A & B.

(2) Now suppose that there is a $1 transaction fee, paid at time 0, for going either long or short the forward contract. Compute the upper and lower no-arbitrage bounds F^- and F^+.

Q11: [Text 8.2]

Suppose that oil forward prices for 1 year, 2 years, and 3 years are \$20, \$21, and \$22. The 1-year effective annual interest rate is 6.0%, the 2-year interest rate is 6.5%, and the 3-year interest rate is 7.0%.

What is the 3-year swap price?

TABLE 8.9

Quarter	1	2	3	4	5	6	7	8
Oil forward price	21	21.1	20.8	20.5	20.2	20	19.9	19.8
Gas swap price	2.2500	2.4236	2.3503	2.2404	2.2326	2.2753	2.2583	2.2044
Zero-coupon bond price	0.9852	0.9701	0.9546	0.9388	0.9231	0.9075	0.8919	0.8763
Euro-denominated zero-coupon bond price	0.9913	0.9825	0.9735	0.9643	0.9551	0.9459	0.9367	0.9274
Euro forward price (\$/€)	0.9056	0.9115	0.9178	0.9244	0.9312	0.9381	0.9452	0.9524

Q12: [Text 8.14.]

Using the zero-coupon bond yields in Table 8.9, what is the fixed rate in a 4-quarter interest rate swap?

Solutions

Q1:

a) A short sale of JKI stock entails borrowing shares of JKI and then selling them, receiving cash, and we learned that we sell assets at the bid price. Therefore, initially, we will receive the proceeds from the sale of the asset at the bid (ignoring the commissions and interest). After 180 days, we cover the short position by buying the JKI stock, and we saw that we will always buy at the ask. Therefore, we earn the following profit:

$$400 \times (\$25.12) - 400 \times (\$23.06) = \$\,824.00$$

b) We have to pay the commission twice. The commission will reduce our profit:

$$400 \times (\$25.12) - 400 \times (\$25.12) \times 0.003 - (400 \times (\$23.06) \cdot (.003)) + 400 \times (\$23.06))$$

$$= 766.184 \quad \text{OR} \quad 400\left[25.12(1-.003) - 23.06(1+.003)\right] = \$766.184$$

c) The proceeds from short sales, minus the commission charge are \$10,017.88 (or \$10,048 if you ignore the commission charge). Since the 6-month interest rate is given, and the period of our short sale is exactly half a year, we can directly calculate the interest we could earn (and that we now lose) on a deposit of \$10,017.88

$$= 10{,}017.88(.03) = \$300.54$$

or, without taking into account the commission charge: $10{,}048\,(.03) = \$301.44$

Q2:

a) Remember that the terminology bid and ask is formulated from the market makers perspective. Therefore, the price at which you can buy is called the ask price. Furthermore, you will have to pay the commission to your broker for the transaction. You pay:

$$(\$41.05 \times 100) + \$20 = \$4,125.00$$

b) Similarly, you can sell at the market maker's bid price. You will again have to pay a commission, and your broker will deduct the commission from the sales price of the shares. You receive:

$$(\$40.95 \times 100) - \$20 = \$4,075.00$$

c) Your round-trip transaction costs amount to:

$$\$4,125.00 - \$4,075.00 = \$50$$

Q3.

(1)

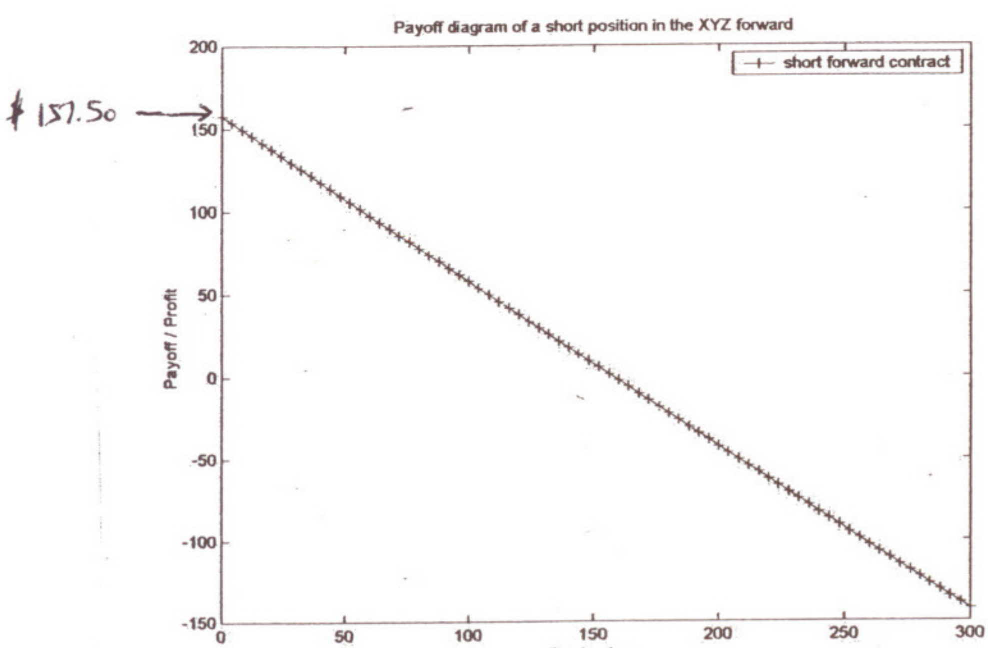
The forward price of \$1,200 is worse for us if we want to buy a forward contract. To understand this, suppose the index after one year is \$1,150. While we have already made money in part a) with a forward price of \$1,100, we are still losing \$50 with the new price of \$1,200. As there was no advantage in buying either stock or forward at a price of \$1,100, we now need to be "bribed" to enter into the forward contract. We somehow need to find an equation that makes the two strategies comparable again. Suppose that we lend some money initially together with entering into the forward contract so that we will receive \$100 after one year. Then, the payoff from our modified forward strategy is: $\$S_T - \$1,200 + \$100 = \$S_T - \$1,100$, which equals the payoff of the "borrow to buy index" strategy. We have found the future value of the premium somebody needs us to pay. We still need to find out what the premium we will receive in one year is worth today. We need to discount it: $\$100/(1+0.10) = \$90.91$.

(2)

Similarly, the forward price of \$1,000 is advantageous for us. As there was no advantage in buying either stock or forward at a price of \$1,100, we now need to "bribe" someone to sell this advantageous forward contract to us. We somehow need to find an equation that makes the two strategies comparable again. Suppose that we borrow some money initially together with entering into the forward contract so that we will have to pay back \$100 after one year. Then, the payoff from our modified forward strategy is: $\$S_T - \$1,000 - \$100 = \$S_T - \$1,100$, which equals the payoff of the "borrow to buy index" strategy. We have found the future value of the premium we need to pay. We still need to find out what this premium we have to pay in one year is worth today. We simply need to discount it: $\$100/(1+0.10) = \$90.91$. We should be willing to pay \$90.91 to enter into the one year forward contract with a forward price of \$1,000.

Q4.

(a) It does not cost anything to enter into a forward contract—as a seller, we do not receive a premium. Therefore, the payoff diagram of a forward contract coincides with the profit diagram. The graphs have the following shape:



Q4(cont'd.):

(b) We can invest the proceeds from the initial short-sale. We do so by lending \$150. After one year we receive: $\$150 \times (1 + 0.05) = \$157.50$. Therefore, our total profit at expiration from the short sale of a stock that was financed by a loan was: $\$157.50 - S_T$, where S_T is the value of one share of MNO at expiration. But this profit from selling the stock and lending the proceeds is the same as the profit from our short forward contract, and none of the positions requires any initial cash—but then, there is no advantage in investing in either instrument.

(c) The owner of the stock is entitled to the dividend. If we borrow an asset from a lender, we have the obligation to make any payments to the lender that she is entitled to as a stockholder. Therefore, as a short seller, we have to pay the dividend. As the seller of a forward contract, we do not have to pay the dividend to our counterparty, because she only has a claim to buy the stock in the future for a given price from us, but she does not own it yet. Therefore, it does matter now whether we short-sell the stock or the sell the forward contract. Because everything else is the same as in part (a) and (b), it is now beneficial to sell the forward contract.

Q5:

This question is a direct application of the Put-Call-Parity. We will use equation (3.1) in the following, and input the given variables:

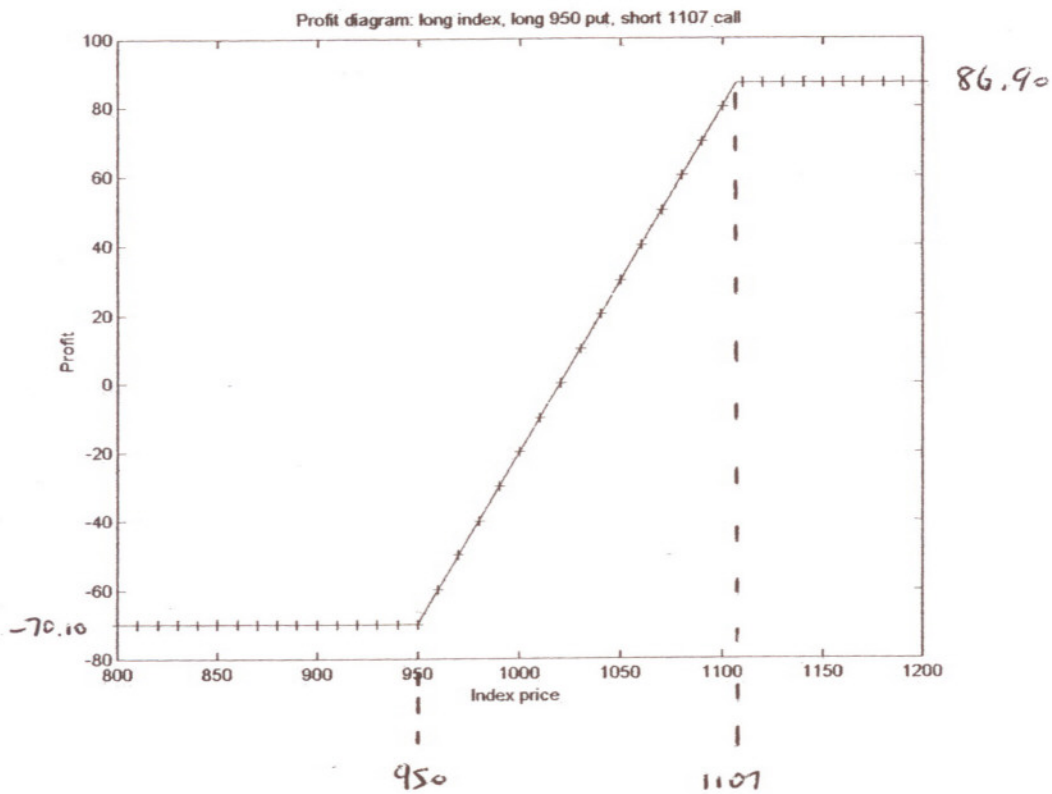
$$
\begin{aligned}
& Call\,(K,t) - Put\,(K,t) = PV\left(F_{0,t} - K\right) \\
\Leftrightarrow\ & Call\,(K,t) - Put\,(K,t) - PV\left(F_{0,t}\right) = -PV\,(K) \\
\Leftrightarrow\ & Call\,(K,t) - Put\,(K,t) - S_0 = -PV\,(K) \\
\Leftrightarrow\ & \$109.20 - \$60.18 - \$1{,}000 = -\frac{K}{1.02} \\
\Leftrightarrow\ & K = \$970.00
\end{aligned}
$$

Q6.

Our initial cash required to put on the collar, i.e. the net option premium, is as follows: $-\$51.873 + \$51.777 = -\$0.096$. Therefore, we receive only 10 cents if we enter into this collar. The position is very close to a zero-cost collar.

The profit diagram looks as follows:

Q6(cont'd):



Alternative Solution:

$T = 0.5 \qquad 1020 = F_{0,T} = (1.02)S_0 \Rightarrow S_0 = 1000$

$$\text{Profit} = [S_T - 1000(1.02)] + [(950 - S_T)_+ - 51.777(1.02)] + [51.873(1.02) - (S_T - 1107)_+]$$

$$= S_T + (950 - S_T)_+ - (S_T - 1107)_+ - 1020.10$$

$$= \begin{cases} -70.10 & S_T \le 950 \\ S_T - 1020.10 & 950 < S_T \le 1107 \\ 86.90 & 1107 < S_T \end{cases}$$

[We now see all values of key points on chart.]

Q7:

a) Expected pre-tax profit

Firm A: $E[\text{Profit}] = 0.5 \times (\$1,000) + 0.5 \times (-\$600) = \$200$
Firm B: $E[\text{Profit}] = 0.5 \times (\$300) + 0.5 \times (\$100) = \$200$

Both firms have the same pre-tax profit.

b) Expected after tax profit.

Firm A:

		bad state	good state
(1)	Pre-Tax Operating Income	–\$600	\$1,000
(2)	Taxable Income	\$0	\$ 1,000
(3)	Tax @ 40%	0	\$400
(3b)	Tax Credit	\$240	0
	After-Tax Income (including Tax credit)	–\$360	\$600

This gives an expected after-tax profit for firm A of:

$$E[\text{Profit}] = 0.5 \times (-\$360) + 0.5 \times (\$600) = \$120$$

Firm B:

		bad state	good state
(1)	Pre-Tax Operating Income	\$100	\$300
(2)	Taxable Income	\$100	\$300
(3)	Tax @ 40%	\$40	\$120
(3b)	Tax Credit	0	0
	After-Tax Income (including Tax credit)	\$60	\$180

This gives an expected after-tax profit for firm B of:

$$E[\text{Profit}] = 0.5 \times (\$60) + 0.5 \times (\$180) = \$120$$

If losses receive full credit for tax losses, the tax code does not have an effect on the expected after-tax profits of firms that have the same expected pre-tax profits, but different cash-flow variability.

Q8: Profit = Revenue − Cost + Hedge Net CF

Revenue = 40,000 (2.50) = 100,000
Cost = 60,000 + 20,000 (1.65) = 93,000
Hedge Net CF = 20,000 [(1.65 − 1.60) + − .12 (1.04)] = −1496
∴ Profit = 100,000 − 93,000 + (−1496) = 5504

Answer = \$5504

["Quick Solution:" 2.50(40,000) − 60,000 − 20,000(1.60) − 20,000(.12)(1.04)
= \$5504.]

Q9.

First, we need to find the fair value of the forward price. We plug the continuously compounded interest rate, the dividends and the time to expiration in years into the valuation formula:

$$
\begin{aligned}
F_{0,T} &= S_0 e^{rT} - \sum_{i=1}^{n} D e^{r\times(T-t_i)} \\
&= \$66e^{0.05\times 0.75} - 0.45\times e^{0.05\times(0.75-0.25_i)} - 0.45\times e^{0.05\times(0.75-0.5_i)} - 0.45\times e^{0.05\times(0.75-0.75_i)} \\
&= \$68.522 - 0.45\times(1.0253 + 1.0126 + 1) \\
&= \$68.522 - 0.45\times(3.0379) \\
&= \$67.155
\end{aligned}
$$

If we observe a forward price of \$67.50, we know that the forward is too expensive, relative to the fair value we have determined. Therefore, we will sell the forward at \$67.50, and create a synthetic forward for \$67.155, making a sure profit of \$0.345. As we sell the real forward, we engage in cash and carry arbitrage:

Description	Today	in 3 Months	in 6 Months	in 9 Months
Short forward	0			$\$67.50 - S_T$
Buy stock	–\$66.00			S_T
Borrow	\$66.00	\$66.00		–\$68.522
Receive 1st dividend, invest dividend		\$0.45 –\$0.45		\$0.4614
Receive 2nd dividend, invest dividend			\$0.45 –\$0.45	\$0.4557
Receive 3rd dividend				\$0.45
TOTAL	0	0	0	\$0.345

This position requires no initial investment, has no MNO price risk, and has a strictly positive payoff. We have exploited the mispricing with a pure arbitrage strategy.

Summary: $F_{0,T} = 67.155$

∴ short forward

AND:

1) Borrow money to purchase stock ($S_0 = 66$)

2) Buy stock with borrowed money.

Then: Explain where the arbitrage comes from. A table is good but can be done other ways.

Q10: Recall the formulas for F^+ and F^- on pages 138 + 139 of the text.

$$F^+ = (S_0^a + 2k)\, e^{r^b T}$$

$$F^- = (S_0^b - 2k)\, e^{r' T}$$

[The trader does not have to trade the stock at time T, only net delivery with the forward + thus only 2k appears.]

(1) $A = F^+ = 800\, e^{.055T}$

IF $T=1$: $A = 845.23$

$B = F^- = 800\, e^{.05T}$

IF $T=1$: $B = 841.02$

[$A = 845.23$
$B = 841.02$]

(2) $F^+ = (800+1)\, e^{.055T}$

$= 801\, e^{.055T}$

$= 846.29$ if $T=1$

[\$1 on forward but still \$0 on stock.]

$F^- = (800-1)\, e^{.05T}$

$= 799\, e^{.05T}$

$= 839.97$ if $T=1$.

[$F^+ = 846.29$
$F^- = 839.97$]

Q11:

We first solve for the present value of the cost per three barrels, based on the forward prices:

$$\frac{\$20}{1.06} + \frac{\$21}{(1.065)^2} + \frac{\$22}{(1.07)^3} = 55.3413.$$

We then obtain the swap price per barrel by solving:

$$\frac{x}{1.06} + \frac{x}{(1.065)^2} + \frac{x}{(1.07)^3} = 55.341$$

$$\Leftrightarrow \quad x = 20.9519$$

Answer = \$20.95.

Q12:

$$1 = cP(0, 1/4) + cP(0, 1/2) + cP(0, 3/4) + cP(0,1) + P(0,1)$$

$$\Rightarrow \quad c = \frac{1 - P(0,1)}{P(0, 1/4) + P(0, 1/2) + P(0, 3/4) + P(0,1)} = .0159$$

$$\therefore \quad R = 1.59\%.$$

[OR (unnecessarily long)

From the given zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Forward interest rate
1	1.0150
2	1.0156
3	1.0162
4	1.0168
5	1.0170
6	1.0172
7	1.0175
8	1.0178

Now, we can calculate the swap prices for 4 and 8 quarters according to the formula:

$$X = \frac{\sum_{i=1}^{n} P_0(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=1}^{n} P_0(0, t_i)}, \text{ where } n = 4 \text{ or } 8$$

This yields the following prices:

4-quarter fixed swap price: 1.59015%]